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HYDROMAGNETIC THERMAL INSTABILITY OF HETROGENEOUS INCOMPRESSIBLE VISCOUS SHEAR FLOW

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Abstract--In this paper, we discuss the effect of a uniform magnetic field applied in the direction of streaming on a steady, thermally, density stratified shear flow of a viscous, incompressible, perfect electrically conducting and heat conducting fluid confined between two non-deformable free horizontal boundaries which are maintained at constant temperatures. We also discuss the necessary condition of instability for R_2 >0 and R_3 <0 and obtained growth rate of unstable modes for R_2>0 and R_3 <0. Further, we have found out that growth rate of unstable modes for large wave numbers and Q<<1 and finally, we derived the sufficient condition of stability for non-oscillatory modes and also find bounds of n_r for unstable modes

Keywords: H, K, n, p, T, u, ρ , κ is magnetic field, wave number, complex constant, pressure, temperature, velocity, density, thermal conductivity of fluid

1.1 Introduction Many researches such as Miles (1961), Banerjee, Dube, Gupta(1975) etc. have studied the effect of stratification on the non-viscous and incompressible flows. Attempts have also been made to investigation the effects of viscosity and thermal conduction on the stability of stratified flows. Rayleigh (1916) in a fundamental research, showed that what decides the stability or otherwise of a fluid layer heated from below is the numerical value of the non-dimensional parameter, ,called Rayleigh number, where g denotes the acceleration due to gravity, d the depth of the layer ,the uniform adverse temperature gradient and are the coefficient of volume expansion , thermal conductivity and Kinematic viscosity , respectively. Rayleigh further showed that instability must set in when R exceeds a certain critical value R_c , a stationary pattern of motions must done to prevail or the principle of exchange of stabilities is valid. Banerjee (1972) has investigated the stability of a continuously stratified layer of viscous, incompressible fluid, statically confined between two horizontal boundaries of different but uniform temperature. The fluid is being heated or cooled from below.

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One of the Principle results established in the paper is a 'Circle Theorem' which units the complex amplification rate of an arbitrary oscillatory mode inside a circle. Sharma and Sharma (1992) studied the thermal instability in Maxwellian visco-elastic fluid in porous medium. They discussed the thermal convection in a layer of Maxvellian visco-elastic fluid heated from below in porous medium an analyzed the effects of uniform rotation and uniform magnetic field for stationary convection. It was found that Maxwellian fluid behaves like a Newtonian fluid. Further, the critical Rayleigh number was found to increase with the increase in magnetic field and rotation.

Juarez and Busse (1995) investigated the stability of spatially periodic solutions for steady and oscillatory two-dimensional convection in a fluid-saturated porous medium. They analyzed the unit where viscous effects are dominant and Darcy's Law can be applied. Galerkin Method solutions that appear in non-linear convection at Rayleigh number up to 20 times the critical value. The stability boundaries for arbitrary, infinitesimal perturbation where obtained. It was also found that at a given Rayleigh number close to the critical values. The stability of this state with respect to infinitesimal perturbations of any wave number was discussed and the resulting temporal dynamics in the resonance of the stable regions was briefly analyzed and used to discuss the stability of density stratified flow through a porous medium. The important results obtained by them include a relation between two and three dimensional disturbances, the stabilizing character of porous media with high Darey's resistance,

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bounds on the complex wave velocity of unstable modes and a number of sufficient conditions for the stability of these systems. Bansal, Jaimala and Agrawal (1999) studied the shear flow instability of an incompressible visco-elastic fluid in porous medium in the presence of a weak magnetic field. They established the sufficient condition for stability and confirm the stabilizing role of magnetic field and fluid viscosity and a destabilizing character of medium permeability, velocity shear and top-heavy density distribution.
In this present chapter, we have studied the effect of a uniform magnetic field applied in the effect of a uniform magnetic field applied in the direction of streaming on a steady, thermally, density stratified shear flow of a viscous, incompressible, perfect electrically conducting and heat conducting fluid confined between two non-deformable free horizontal boundaries which are maintained at constant temperatures.

1.2 Formulation of the Problem:

For an incompressible, viscous, heterogeneous, heat conducting fluid of infinite electrical conductivity, the equations of magnetohydrodynamics with the initial state is therefore one in which the velocity, the magnetic field, temperature, density and pressure at any point in the fluid region are given by

$$\begin{bmatrix} u_i = [U(y),0,0] \\ H_j = [H,0,0] \\ T = T_0 + \beta y \\ \rho = \rho_0[f(y) - \alpha\beta y] \end{bmatrix} \quad (1)$$

Pressure = p
Where β is the uniform temperature gradient, which is maintained, ρ_0 is the density at the lower boundary and α is the co-efficient of volume expansion of the fluid.

Let the initial state be slightly perturbed so that the perturbed state are given by

$$\begin{bmatrix} \overset{*}{u} = (U+u, v, 0) \\ \overset{*}{H} = (H+h_x, h_y, 0) \\ \overset{*}{T} = T+\theta \\ \overset{*}{\rho} = \rho_0[f(y) + \frac{\delta\rho}{\rho_0} + \alpha(T_0 - T + \theta) \\ \text{and} \\ \overset{*}{p} = p + \delta p \end{bmatrix} \quad (2)$$

Where $(u,v,0), (h_x, h_y, 0), \theta, \delta p$ and $\delta\rho$ are respectively the perturbations in the velocity field, magnetic field, temperature, pressure and density.
Now we analyzing the disturbances into normal modes, we seek solutions whose dependence on x ,y and t is given by

$$f(x,y,t) = f(y)\exp[i(Kx + \sigma t)] \quad (3)$$

where K is the wave number of the disturbance and σ is a constant, which is in general complex. Hence we get

$$(U-n)(D^2-a^2)v + i\frac{P}{a}(D^2-a^2)^2 v - (D^2U)v = Q(D^2-a^2)\frac{v}{(U-n)} + iR_2 a\theta + R_3\frac{v}{(U-n)} \quad (4)$$

and

$$R_1(D^2-a^2)\theta - ia(U-n)\theta = v \quad (5)$$

where $Y = dy$, $R_1 = \frac{\kappa}{Ud}$, $Q = \frac{\mu_e H^2}{4\pi\rho_0 U^2}$

$D = \frac{1}{d}D$, $R_2 = \frac{g\alpha\beta d}{U^2}$, $\sigma = -\frac{naU}{d}$

$a = Kd$, $P = \frac{\mu}{\rho_0 Ud}$, $R_3 = \frac{gd^2(Df)}{U^2}$, $\theta = \beta\theta$

Where d is the distance between the plates.
The boundary conditions are given by

$$\text{and}\begin{Bmatrix} v = 0 = \theta \text{ at } Y = 0 \text{ and } Y = 1 \\ D^2 v = 0 \text{ at } Y = 0 \text{ and } Y = 1 \end{Bmatrix} \text{(For free boundaries)} \quad (6)$$

Thus for a given a, P, R_1, R_2, R_3 and Q, equations (4) and (5) together with the boundary conditions (6) present an Eigen value problem for n and the system is stable , neutral or unstable according as the imaginary part of n , namely n_i is negative , zero or positive.
Further,

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(i) For non-adverse temperature gradient, $\beta > 0$ and so $R_2 > 0$;

(ii) For adverse temperature gradient , $\beta < 0$ and so $R_2 < 0$;

(iii) For stable density stratification , $Df < 0$ everywhere and so $R_3 < 0$

(iv) For unstable density stratification, $Df > 0$ and so $R_3 > 0$.

1.3 A NECESSARY CONDITION OF INSTABILITY FOR $R_2>0$ AND $R_3<0$

Taking the complex conjugate of equation (5) multiplying throughout by θ, integrating over the range of Y and using the boundary conditions (6), we get

$$-R_1\int\left[|D\theta|^2+a^2|\theta|^2\right]dy+ia\int\left(U-\overset{*}{n}\right)|\theta|^2dy=\int\overset{*}{v}\theta dy$$

Or

$$-R_1\int\phi_1dy+ia\int\left(U-\overset{*}{n}\right)|\theta|^2dy=\int\overset{*}{v}\theta dy \qquad (7)$$

Where $\phi_1=\left[|D\theta|^2+a^2|\theta|^2\right]>0$ (8)

Now multiplying equation (4) throughout by $\overset{*}{v}$ (the complex conjugate of v), integrating over the range of Y, using the boundary conditions (6) and putting for $\int\overset{*}{v}\theta dy$ from equation (7), we get

$$\int\overset{*}{v}(U-n)(D^2-a^2)vdy+\frac{iP}{a}\int\overset{*}{v}(D^2-a^2)^2vdy$$
$$-\int(D^2U)|v|^2dy=Q\int\overset{*}{v}\left(D^2-a^2\right)\frac{v}{(U-n)}dy$$
$$+iR_2a[-R_1]\phi_1dy+ia\int(U-\overset{*}{n})|\theta|^2dy+R_3\int\frac{|v|^2}{(U-n)}dy$$

or

$$\int\overset{*}{v}UD^2Vdy-a^2\int U|v|^2dy+n\int Q_1dy+i\frac{P}{a}\int Q_2dy$$
$$-\int\left(D^2U\right)|v|^2dy-Q\int\overset{*}{v}D^2\left(\frac{v}{U-n}\right)dy$$
$$+a^2Q\int\frac{|v|^2}{(U-n)}dy+iR_1R_2a\int\phi_1dy+R_2a^2\int(U-\overset{*}{n})|\theta|^2dy$$
$$-R_3\int\frac{|v|^2}{(U-n)}dy=0$$

(9)

Where

$$\text{And}\begin{cases}Q_1=\left[|Dv|^2+a^2|v|^2\right]>0\\ Q_2=\left[\left|D^2v\right|^2+2a^2|Dv|^2+a^4|v|^2\right]>0\end{cases} \qquad (10)$$

now

$$\int\overset{*}{v}U(D^2v)dy=-\int U|Dv|^2dy-\int(DU)\overset{*}{v}(Dv)dy$$

and

$$-Q_1\int\overset{*}{v}D^2\left(\frac{v}{U-n}\right)dy+a^2Q\int\frac{|v|^2}{(U-n)}dy$$
$$=Q\int\frac{Q_1}{(U-n)}dy-Q\int\frac{(DU)v(D\overset{*}{v})}{(U-n)^2}dy$$

Thus, equation (9) becomes

$$-\int UQ_1dy-\int(DU)\overset{*}{v}Dvdy+n\int Q_1dy+i\frac{P}{a}\int Q_2dy$$
$$-\int(D^2U)|v|^2dy+Q\int\frac{Q_1}{(U-n)}dy$$
$$-Q\int\frac{(DU)vD\overset{*}{v}}{(U-n)^2}dy+iR_1R_2a\int\phi_1dy+R_2a^2\int(U-\overset{*}{n})|\theta|^2dy$$
$$-R_3\int\frac{|v|^2}{(U-n)}dy=0$$

(11)

Taking the imaginary part of this equation, we get

$$-\frac{q}{2a}\int Q_1dy+n_i\int Q_1dy+\frac{P}{a}\int\left[\left|D^2v\right|^2+a^2|Dv|^2\right]dy$$
$$+aP\int Q_1dy+n_iQ\int\frac{Q_1}{|U-n|^2}dy$$
$$-\frac{qQ}{2a}\int\frac{Q_1}{|U-n|^2}dy+R_1R_2a\int\phi_1dy+R_2a^2n_i\int|\theta|^2dy$$
$$-n_iR_3\int\frac{|v|^2}{|U-n|^2}dy\le 0$$

(12)

Where we have used that

$$\text{Im}\int(DU)\overset{*}{v}(Dv)dy\le\frac{q}{2a}\int Q_1dy,$$

$$\text{Im}\int\frac{(DU)vD\overset{*}{v}}{(U-n)^2}dy\le\frac{q}{2a}\int\frac{Q_1}{|U-n|^2}dy,$$

And $q=\max.|DU|$

Inequality (12) can be written as

$$\int\left[aP-\frac{q}{2a}-\frac{qQ}{2a|U-n|^2}\right]Q_1dy+\frac{P}{a}\int\left[\left|D^2v\right|^2+a^2|Dv|^2\right]dy$$
$$+R_1R_2a\int\phi_1dy+n_i\int\left[1+\frac{Q}{|U-n|^2}\right]Q_1+R_2a^2|\theta|^2$$
$$-\frac{R_3|v|^2}{|U-n|^2}dy\le 0 \qquad (13)$$

From inequality (13), we that for n_i>0 i.e for unstable modes for R_2>0, R_3<0, the necessary condition of instability is given by

$$aP-\frac{q}{2a}-\frac{qQ}{2a|U-n|^2}<0 \qquad (14)$$

At least at one point in the interval [0,1]. Inequality (14) can be written as

$$\left(U_{Y_s}-n_r\right)^2+n_i^2<\frac{qQ}{2P\left(a^2-\frac{a}{2P}\right)} \qquad (15)$$

For $a^2>\frac{q}{2P}$ and $0<Y_S<1$, which shows that the complex wave velocity of unstable modes lie Inside the circle given by (15) . Further, from (14) the wave number range which may be unstable is given by

$$a^2<\frac{q}{2P}\left[1+\frac{Q}{|U-n|^2}\right] \qquad (16)$$

Again from inequality (13), we find that under the condition

$$aP-\frac{q}{2a}-\frac{qQ}{2a|U-n|^2}\ge 0 \qquad (17)$$

Everywhere in the flow region, n_i must be negative, which means that the wave number range given by

$$a^2\ge\frac{q}{2P}\left[1+\frac{Q}{|U-n|^2}\right] \qquad (18)$$

will be a stable wave number range.
In the absence of magnetic field ie. (Q=0), inequality (13) reduces to

$$\int\left[aP-\frac{q}{2a}\right]Q_1dy+\frac{P}{a}\int\left[\left|D^2v\right|^2+a^2|Dv|^2\right]dy$$
$$+R_1R_2a\int\phi dy+n_i\int\left[Q_1+R_2a^2|\theta|^2-R_3\frac{|v|^2}{|U-n|^2}\right]dy\le 0 \qquad (19)$$

Therefore, for R_3<0, R_2 >0 and $n_i>0$, the inequality (43) gives that $(a^2P-\frac{1}{2}q)$ should be negative at least at one point in the interval [0,1]. Thus, the wave number range which may be unstable is given by

$$a^2<\frac{q}{2P} \qquad (20)$$

Further, from inequality (19), we also conclude that under the condition $\left(a^2P-\frac{1}{2}q\right)\ge 0$, everywhere in the flow region, $n_i<0$. Thus the wave number range

$$a^2\ge\frac{q}{2P} \qquad (21)$$

will be stable wave number range.

Now if we compare (21) with (16), then we find the magnetic field decreases with the range of stable wave numbers, showing that its destabilizing character. We further conclude that the wave number range given by

$$\frac{q}{2P}<a^2<\frac{q}{2P}\left[1+\frac{Q}{|U-n|^2}\right] \qquad (22)$$

May be unstable and if unstable modes exist then they will lie inside the circle (15). This is another wave number range which may contain unstable modes, is entirely due to the presence of the magnetic field and in this sense the magnetic field has a destabilizing character.

1.4 GROWTH RATE OF UNSTABLE MODES FOR R_2>0 AND R_3<0

Inequality (13) can be written as

$$\int\left[\frac{q}{2a}+\frac{qQ}{2a|U-n|^2}-aP-n_i-\frac{n_iQ}{|U-n|^2}\right]Q_1dy$$
$$+n_iR_3\int\frac{|v|^2}{|U-n|^2}dy-\frac{P}{a}\int\left[\left|D^2v\right|^2+a^2|Dv|^2\right]dy$$
$$\ge R_1R_2a\int\phi_1dy+R_2a^2n_i\int|\theta|^2dy \qquad (23)$$

From inequality (24), we find that the presence of viscous term decreases the upper bound of the growth rate and thus the viscosity has a stabilizing

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role. Further, Q occurs in the denominator and with its increase the value of the term,

$$\left[\frac{a^2P}{1+\frac{Q}{|U-n|^2}}\right]$$

Decreases and in this sense the magnetic field has a destabilizing effect.

1.5 GROWTH RATE OF UNSTABLE MODES FOR LARGE WAVE NUMBERS AND Q<<1

For weak magnetic field (Q<<1) and for large wave numbers, the term $QD^2\left(\frac{v}{U-n}\right)$ can be neglected in comparison to $Qa^2\left(\frac{v}{U-n}\right)$ in equation (4) and in that case equation (9) becomes

$$\int \overset{*}{v} UD^2Vdy - a^2\int U|v|^2dy + \overset{*}{n}\int Q_1dy + i\frac{P}{a}\int Q_2dy - \int (D^2U)|v|^2dy - \int (R_3 - a^2Q)\frac{|v|^2}{(U-n)}dy + iR_1R_2a\int \phi_1 dy + R_2a^2\int (U-\overset{*}{n})|\theta|^2dy = 0 \qquad (25)$$

Taking the imaginary part of equation (25), we get

$$\int\left[\frac{q}{2a} - n_i - aP\right]|Dv|^2dy + \int\left[a^2\left(\frac{q}{2a} - n_i - aP\right) - \frac{(a^2Q - R_3)n_i}{|U-n|^2}\right]|v|^2dy - \frac{P}{a}\int\left[|D^2v|^2 + a^2|Dv|^2\right]dy \ge R_1R_2a\int\phi_1 dy + R_2a^2n_i\int|\theta|^2dy \qquad (26)$$

Now for $R_2>0$, $R_3<0$ and $n_i > 0$ (unstable modes), from inequality (26), we have

$$\left[\frac{q}{2a} - n_i - aP\right] - \frac{(a^2Q - R_3)n_i}{a^2|U-n|^2} > 0$$

must necessarily hold for an arbitrary unstable mode i.e.

$$an_i < \frac{(q - 2a^2P)}{2\left[1 + \frac{(a^2Q - R_3)}{a^2|U-n|^2}\right]} \qquad (27)$$

Inequality (27) gives the rate of growth of unstable modes for the case when we applied the weak magnetic field i.e Q<<1 and the wave number a is large. In this case we find the stabilizing role of magnetic field and viscosity.

1.6 SUFFICIENT CONDITION OF STABILITY FOR NON-OSCILLATORY MODES

In the subsequent analysis we wish to investigate a sufficient condition of stability for non-oscillatory modes, when the viscosity of the fluid is small. Under this approximation, we can neglect the term PD^4V in comparison to the other terms (Since V is a perturbation quantity and we assume that the perturbations are arbitrary small).Further, for non-oscillatory modes, $n_r = 0$ and so $n = in_i$ only. Thus for non-oscillatory modes and for $P << 1$, equations (4) and (5) become

$$(U - in_i)(D^2 - a^2)v - 2iaP(D^2 - a^2)v - ia^3Pv - (D^2U)v - Q(D^2 - a^2)\frac{v}{(U - in_i)} - iR_2a\theta - R_3\left(\frac{v}{(U - in_i)}\right) = 0 \qquad (28)$$

and

$$R_1(D^2 - a^2)\theta - ia(U - in_i)\theta = v \qquad (29)$$

Now using the transformations

$$V = (U - in_i)F \quad \text{and} \quad \theta = (U + in_i)\phi \qquad (30)$$

In equation (28) and (29), we get

$$(U - in_i)(D^2 - a^2)[(U - in_i)F] - 2iaP(D^2 - a^2)[(U - in_i)F] - ia^3P[(U - in_i)F] - (D^2U)[(U - in_i)F] - Q(D^2 - a^2)F - iR_2a[(U + in_i)\phi] - R_3F = 0 \qquad (31)$$

and

$$R_1(D^2 - a^2)[(U + in_i)\phi] - ia(U - in_i)[(U + in_i)\phi] = (U - in_i)F \qquad (32)$$

The boundary conditions are given by

$$F = 0 = \phi \quad \text{at } Y = 0 \text{ and } Y = 1$$

and $D^2F = 0$ at $Y = 0$ and $Y = 1$ (for free boundary) (33)

Now taking the complex conjugate of equation (32), multiplying throughout by ϕ, integrating over

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the range of Y and using the boundary conditions (33), we get

$$R_1\int\phi(D^2-a^2)\left[(U-in_i)\overset{*}{\phi}\right]dy+ia\int(U^2+n_i^2)|\phi|^2dy$$

$$=\int(U+in_i)\overset{*}{F}\phi dy$$

or

$$-R_1\int(U-in_i)\left[|D\phi|^2+a^2|\phi|^2\right]dy-R_1\int(DU)(D\phi)\overset{*}{\phi}dy$$

$$+ia\int(U^2+n_i^2)|\phi|^2dy=\int(U+in_i)\overset{*}{F}\phi dy \qquad (34)$$

Now equation (31) can be written as

$$D\left[(U-in_i)^2DF\right]-a^2(U-in_i)^2F$$
$$-\frac{2iaP}{(U-in_i)}D\left[(U-in_i)^2DF\right]-2iaP\left(D^2U\right)F$$
$$+ia^3P(U-in_i)F-Q\left(D^2-a^2\right)F$$
$$-iR_2a(U+in_i)\phi-R_3F=0$$
$$(35)$$

Multiplying (35) by throughout $\overset{*}{F}$ and integrating over the range of Y and using the boundary conditions (33) and putting for $\int(U+in_i)\overset{*}{F}\phi dy$ From equation (34),we get

$$\int\overset{*}{F}D\left[(U-in_i)^2DF\right]dy-a^2\int(U-in_i)^2|F|^2dy$$
$$-2iaP\int\frac{\overset{*}{F}}{(U-in_i)}D\left[(U-in_i)^2DF\right]dy-2iaP\int\left(D^2U\right)|F|^2dy$$
$$+ia^3P\int(U-in_i)|F|^2dy-Q\int\overset{*}{F}\left(D^2-a^2\right)Fdy$$
$$-iR_2a\left[-R_1\int(U-in_i)\left[|D\phi|^2+a^2|\phi|^2\right]dy\right]$$
$$R_1\int(DU)(D\phi)D\overset{*}{\phi}dy+ia\int\left(U^2+n_i^2\right)|\phi|^2dy-R_3\int|F|^2dy=0$$
$$(36)$$

For linear velocity profile, $D^2U=0 \Rightarrow DU=$constant, and therefore equation (36) becomes

$$\int(U-in_i)^2Q_0dy+2iaP\int\frac{\overset{*}{F}}{(U-i\eta_i)}D\left[(U-in_i)^2DF\right]dy$$
$$-ia^3P\int(U-in_i)|F|^2dy-Q\int Q_0dy$$
$$-iR_1R_2a\int(U-in_i)\phi_0dy-iR_1R_2a(DU)\int(D\phi)\overset{*}{\phi}dy$$
$$-R_2a^2\int\left(U^2+n_i^2\right)|\phi|^2dy+R_3\int|F|^2dy=0$$
$$(37)$$

Where

$$Q_0=\left[|DF|^2+a^2|F|^2\right]>0 \text{ and}$$

$$\phi_0=\left[|D\phi|^2+a^2|\phi|^2\right]>0 \qquad (38)$$

Taking the real part of equation (37), we get

$$\int\left(U^2-n_i^2\right)Q_0dy-2aPn_i\int|DF|^2dy+P(DU)\int Q_0dy$$
$$-a^3Pn_i\int|F|^2dy-Q\int Q_0dy$$
$$-R_1R_2an_i\int\phi_0dy+R_1R_2(DU)\int\phi_0dy$$
$$-R_2a^2\int\left(U^2+n_i^2\right)|\phi|^2dy+R_3\int|F|^2dy\ge 0$$

or

$$\int\left[U^2+P(DU)-Q\right]Q_0dy+R_1R_2(DU)\int\phi_0dy$$
$$-R_2a^2\int U^2|\phi|^2dy+R_3\int|F|^2dy-n_i^2\int Q_0dy$$
$$-R_2a^2n_i^2\int|\phi|^2dy-2aPn_i\int|DF|^2dy$$
$$-a^3Pn_i\int|F|^2dy-R_1R_2an_i\int\phi_0dy\ge 0$$
$$(39)$$

Now for $R_2>0$ and $R_3<0$, from inequality (39), we find that under the condition

$$DU<0 \quad\text{and}\quad U^2+P(DU)-Q<0 \qquad (40)$$

Everywhere in the flow region, n_i cannot be greater than or equal to zero, consequently $n_i<0$ is the only possibility under the condition (40).Thus, the sufficient conditions of stability for non-oscillatory modes are given by

$$\text{and}\begin{bmatrix}DU<0\\ Q>U^2{}_{max}+P(DU)\end{bmatrix} \qquad (41)$$

Further, for $R_2>0$ and $R_3>0$, inequality (39) can be written as

$$\int[U^2+P(DU)-Q]|DF|^2dy+$$
$$\int\left[a^2\left(U^2+P(DU)-Q+\frac{R_3}{a^2}\right)\right]|F|^2dy+R_1R_2(DU)\int\phi_0dy$$
$$-R_2a^2\int U^2|\phi|^2dy-n_i^2\int\phi_0dy-R_2a^2n_i^2\int|\phi|^2dy$$
$$-2aPn_i\int|DF|^2dy-a^3Pn_i\int|F|^2dy-R_1R_2an_i\int\phi_0dy\ge 0$$
$$(42)$$

Now from (42), we find that under the conditions

$$DU<0$$

$$\text{and}\ \ U^2{}_{max}+P(DU)-Q+\frac{R_3}{a^2}<0 \qquad (43)$$

The system cannot be unstable with respect to non-oscillatory modes. In other words, from (43),
We can say that for non-oscillatory modes, the system is stable only for the wave number given by

$$a^2 > \frac{R_3}{\left[Q - U^2_{max} - P(DU)\right]} \quad (44)$$

Where $DU < 0$ and $Q - U^2_{max} - P(DU) > 0$

Thus, we conclude that in the case $R_2>0$ and $R_3<0$, the system is stable for all wave number while in the case of $R_2>0$ and $R_3>0$, the system is stable only for the wave number range given by (44).The stabilizing character of the magnetic field and viscosity can also be seen from (44).

1.7 Bounds of n_r For Unstable Modes

Finally, we investigates the bounds of n_r for unstable modes, we assume that the viscosity of fluid is small and applied the weak magnetic field. Under these approximation, equations (4) and (5) becomes

$$(U-n)(D^2-a^2)v + iPa^3v - (D^2U)v + a^2Q\left(\frac{v}{U-n}\right)$$
$$-iR_2a\theta - R_3\left(\frac{v}{U-n}\right) = 0 \quad (45)$$

and

$$-ia(U-n)\theta = v \quad (46)$$

Eliminating θ between equations (45) and (46), we get

$$(U-n)\left(D^2-a^2\right)v + iPa^3v - \left(D^2U\right)v + a^2Q\left(\frac{v}{U-n}\right)$$
$$+\left(R_2-R_3\right)\left(\frac{v}{U-n}\right) = 0 \quad (47)$$

or

$$(U-n)^2D^2v - a^2(U-n)^2v + iPa^3(U-n)v$$
$$-\left(D^2U\right)(U-n)v + \left(a^2Q + R_2 - R_3\right)v = 0$$

or

$$\left[\left((U-n_r)^2 - n_i^2\right) - 2in_i(U-n_r)\right]D^2v$$
$$-a^2\left[\left((U-n_r)^2 - n_i^2\right) - 2in_i(U-n_r)\right]v$$
$$+iPa^3\left[(U-n_r) - in_i\right]v - \left(D^2U\right)\left[(U-n_r) - in_i\right]v$$
$$+\left(a^2Q + R_2 - R_3\right)v = 0$$

(48)

Multiplying equation (48) throughout by $\left[\left((U-n_r)^2 - n_i^2\right) + 2in_i(U-n_r)\right]\overset{*}{v}$, integrating and using the boundary conditions $\left[v = D^2v = 0 \text{ at } y = 0 \text{ and } y = 1\right]$, we get

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0 \quad (49)$$

Where I_1, I_2, I_3, I_4 and I_5 are define in equation (48)

Now equating the real part of both sides of equation (49), using the fact that $an_i \le q/2$ and for $I = \int F(y)\overset{*}{v}D^2v\,dy$, where $F(y)$ is a real valued continuous of y having continuous derivatives at least up to second order $\operatorname{Re} I = \frac{1}{2}\int\left(D^2F\right)|v|^2dy - \int F|Dv|^2dy$, we have after a little calculation

$$-\int\left[\left((U-n_r)^2 - n_i^2\right)^2 + 4n_i^2(U-n_r)^2\right]$$
$$-n_i^2\left(a^2Q + R_2 - R_3\right)\int|v|^2dy +$$
$$\int\left[(U-n_r)^2\left(6(DU)^2 + \left(D^2U\right)(U-n_r) + \frac{1}{2}qa^2P + a^2Q + R_2 - R_3\right)\right]$$
$$+n_i^2\left(2(DU)^2 + \left(D^2U\right)(U-n_r) + \frac{1}{2}qa^2P\right)|v|^2dy \ge 0$$

(50)

Now from inequality (50), it is clear that for an unstable modes exist if the expression

$$\left[6(DU)^2 + \left(D^2U\right)(U-n_r) + \frac{1}{2}qa^2P + a^2Q + R_2 - R_3\right]$$

is negative, everywhere in the flow region, then the relation (50) cannot be satisfied. Therefore, a necessary condition for the existence of unstable modes is that the expression

$$\left[6(DU)^2 + \left(D^2U\right)(U-n_r) + \frac{1}{2}qa^2P + a^2Q + R_2 - R_3\right]$$

(51)

Is positive

at least at one point in the flow domain.

Now for $D^2U > 0$ everywhere in the flow domain then from inequality (51), we have

$$n_r < U_{max} + \max\left[\frac{6(DU)^2 + \frac{1}{2}qa^2P + a^2Q + R_2 - R_3}{D^2U}\right]$$

(52)

And for $D^2U < 0$ everywhere in the flow domain then from inequality (51), we have

$$n_r > U_{min} - \max\left[\frac{6(DU)^2 + \frac{1}{2}qa^2P + a^2Q + R_2 - R_3}{\left|D^2U\right|}\right]$$

(53)

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Thus for $D^2U > 0$ everywhere in flow domain, n_r is bounded above and for $D^2U > 0$ everywhere in flow domain, n_r is bounded below.

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